SOLUCIÓN DEL TEST DEL 19-6-00

1) b)

$c(D)=1+D+D^6$ polinomio conexiones $\rightarrow$ primitivo $\Rightarrow \mathrm{L}=2^6-1=63$

$p(D)=1+D^2+D^4$ polinomio estados $63\cdot 2=126$

$63\cdot 3=189$

$(c)\rightarrow 189\Rightarrow p(D)=1+D^2+D^4\neq 1+D+D^4 \Rightarrow$ No

$(a),(b)\rightarrow 126\Rightarrow p^{126}(D)=1+D^2+D^4=\mathrm{p}^0(D)$

$\mathrm{D}\cdot\mathrm{p}^{\mathrm{n}}(D)\ \bmod \mathrm{c(D)}=\mathrm{p}^{(\mathrm{n}+1)}(D)$

$(a)\ 125\rightarrow \mathrm{p}^{125}(0)=1+D^2+D^4+D^5\rightarrow ¿\,p^{126}\,?$

$\mathrm{D}\cdot\left(1+\mathrm{D}^2+D^4+D^5\right)\bmod \mathrm{c(D)}=\mathrm{p}^{126}(D)$

$\mathrm{D}^6+D^5+D^3+D \ \big|\ \underline{\mathrm{D}^6+D+1}$

$\underline{\mathrm{D}^6 \quad +\mathrm{D}\ +1}\quad 1$

$\mathrm{D}^5+\mathrm{D}^3+1\neq \mathrm{p}^{126}(D)\quad\Rightarrow$ No

$(b)\ p^{124}(D)=D^5+D^4+D^2\equiv 001011$

$\mathrm{D}\cdot\mathrm{p}^{124}(D)\bmod \mathrm{c(D)}=\mathrm{p}^{125}(D)$ $\quad \mathrm{D}^6+D^5+D^3\ \big|\ \underline{\mathrm{D}^6+D+1}$

$\underline{\mathrm{D}^6+D+1}\quad 1$

$\mathrm{D}^5+D^3+D+1=\mathrm{p}^{125}(D)$

$\mathrm{D}\cdot\mathrm{p}^{125}(D)\bmod \mathrm{c(D)}=\mathrm{p}^{126}(D)$ $\quad \mathrm{D}^6+D^4+D^2+D\ \big|\ \underline{D^6+D+1}$

$\underline{\mathrm{D}^6+D+1}\quad 1$

$\mathrm{D}^4+D^2+1=\mathrm{p}^{126}(D)\Rightarrow$ Sí

2) c)

$x(n)=\{0.1,1.3,0.17\}$

$c(n)=\{-0.06,0.97,-0.1\}$

$$
\begin{array}{lccccc}
h(n)=x(n)*c(n)= & -0.06 & 0.097 & -0.01 & & \\
 & & -0.078 & 1.261 & -0.13 & \\
 & & & -0.0102 & 0.1649 & -0.017 \\
\hline
h(n)=\{ & -0.006 & 0.019 & 1.2408 & 0.0349 & -0.017\} \\
 & & & \| & & \\
 & & & h(0) & &
\end{array}
$$

(a) No, pues h(n) sería: _010_

(b) No lo puedo afirmar, no conozco $\sigma_\eta^2 \rightarrow$ No puedo saber c(n).

$$(c)\ \mathrm{DCM_i} = \frac{0.1^2 + 0.17^2}{1.3^2} = 0.23$$

$$\mathrm{DCM_f} = \frac{-0.006^2 + 0.019^2 + 0.0349^2 + 0.017^2}{1.2408^2} = 0.0012367$$

$$10 \cdot \log(\frac{0.023}{0.0012367}) = 12.7 \text{ dB Sí}$$

3) b)

$$\mathrm{x(n)} = \{0.12, 1.5, 0.27\}$$

$$\mathrm{c(n)} = \{-0.06, 0.97, -0.1\}$$

$$\mathrm{h(n) = x(n) * c(n)} \Rightarrow \quad \mathrm{h(0) = c(-1) \cdot x(1) + c(0) \cdot x(0) + c(1) \cdot x(-1)} = 1.4628$$

$$\mathrm{FAR} = \frac{\sum_i \mathrm{C_i^2}}{\mathrm{h^2(0)}} = \frac{0.06^2 + 0.97^2 + 0.1^2}{1.4628^2} = 0.4689$$

4) b)

Hay que hallar la entropía de la fuente ($\overline{\mathrm{L}} \geq \mathrm{H}$) y multiplicarla por 1000

$$\mathrm{H} = \sum_i \mathrm{p_i} \cdot \log_2\left(\frac{1}{p_i}\right)$$

$$\mathrm{H} = 2 \cdot \frac{1}{36} \cdot \frac{1}{\log_{10}(2)} \cdot \left[\log(36) + 2 \cdot \log\left(\frac{36}{2}\right) + 3\log\left(\frac{36}{3}\right) + 4 \cdot \log\left(\frac{36}{4}\right) + 5 \cdot \log\left(\frac{36}{5}\right) + 3 \cdot \log(6)\right]$$

$$= 3.2744 \text{ bits}$$

$$\mathrm{N} = 1000 \cdot \mathrm{H} = 3274 \text{ bits}$$

5) a)

$$\mathrm{DCM} = \frac{1}{\mathrm{T \cdot x^2(0)}} \cdot \int_{\frac{-1}{2T}}^{\frac{1}{2T}} \left|\sum X(f - \tfrac{n}{T})\right|^2 df - 1$$

$$x(0) = \int_{-\infty}^{\infty} x(f) df = \int_{\frac{-1}{T}}^{\frac{1}{T}} 1 \cdot df = \frac{2}{T} \cdot 1 = \frac{2}{T}$$

$$\int_{\frac{-1}{2T}}^{\frac{1}{2T}} \left|\sum x(f - \tfrac{n}{T})\right|^2 df = 2 \cdot \frac{1}{2 \cdot T} \cdot (2)^2 = \frac{4}{T}$$

$$\mathrm{DCM} = \frac{1}{\mathrm{T} \cdot (\frac{2}{\mathrm{T}})^2} \cdot \frac{4}{T} - 1 = \frac{4T^2}{4T^2} - 1 = 1 - 1 = 0$$

6) b)

$$\mathrm{Pe_{bit}} = \frac{1}{4}\cdot\frac{1}{2}\cdot(Q+2\cdot Q+2\cdot Q+Q) = \frac{6}{8}\cdot Q\left(\frac{d}{s}\right) = \frac{3}{4}\cdot Q\left(\frac{d}{s}\right)$$

$\downarrow$ $\mapsto$ 2 bits/símbolo

4 símbolos equiprobables

$$\mathrm{Pe_{bit}} = \frac{\#\text{bits malos}}{\#\text{bits totales}} = \frac{\#\text{símbolos malos}}{2\cdot\#\text{símbolos totales}} = \frac{1}{2}\cdot \mathrm{Pe_{símbolo}}$$

$\downarrow$

SIGRAY : 2 bits/símbolo

7) b)

$\mathrm{Y} = \mathrm{Y_K} + h$

$\downarrow$ $\mapsto$ si x(1) = -0.7, x(0) = 0, x(1) = 0.7

(0.8 1.1 0.8 1.1) con las secuencias 11 y -1-1

11 $\Rightarrow$ -0.7 0 0.7

-0.7 0 0.7

$\mathrm{Y_K}$ = -0.7 -0.7 0.7 0.7

Y = 0.8 1.1 0.8 1.1 $\quad \mathrm{Y} = \mathrm{Y_K} + h$

$h = \mathrm{Y} - \mathrm{Y_K} = (1.5\ 1.8\ 0.1\ 0.4)$

-1-1 $\Rightarrow$ 0.7 0 -0.7

0.7 0 -0.7

$\mathrm{Y_K}$ = 0.7 0.7 -0.7 -0.7

Y = 0.8 1.1 0.8 1.1

$h = \mathrm{Y} - \mathrm{Y_K} = (0.1\ 0.4\ 1.5\ 1.8)$ $\Rightarrow$ Son las mismas muestras en distinto orden.

Como son independientes $\Rightarrow$ ambas secuencias son igual de verosímiles.

8) d)

a) $\underset{A=8}{\mathrm{QAM\text{-}64}} \Rightarrow \mathrm{E} = \frac{\log_2(A)^2\cdot\frac{1}{T}}{\frac{1+\acute{a}}{T}} = \frac{2\cdot\log_2(A)}{1-\acute{a}} = \frac{2\cdot\log_2(8)}{1+0.75} = \frac{6}{1.75} = 3.4286\,\frac{\mathrm{bps}}{\mathrm{Hz}} \Rightarrow$ No

b) Para $\begin{cases}\mathrm{PAM\text{-}A}\\ \mathrm{QAM\text{-}A^2}\end{cases} \rightarrow\ \mathrm{E} = \frac{2\cdot\log_2(A)}{1-\acute{a}}$ Para $\mathrm{E'} = 2\cdot\mathrm{E} \Rightarrow \mathrm{A'} = \mathrm{A}^2$

c) $\mathrm{PAM\text{-}A} \rightarrow \mathrm{P_E} = 2\cdot\left(1-\frac{1}{A}\right)\cdot Q\left[\sqrt{\left(\frac{3\cdot(1+a)}{A^2-1}\cdot\frac{S}{N}\right)}\right]$ Si $\mathrm{A'} = 2\mathrm{A} \Rightarrow \frac{\mathrm{S'}}{\mathrm{N}} = 4\frac{S}{N} \Rightarrow +6\mathrm{dB} \Rightarrow$ No

9) b)

$x \in \{1,2,3,4,5,6\}$

Si cara $\left(p = \frac{1}{2}\right)$

x	x mod 3
1	1
2	2
3	0
4	1
5	2
6	0

$\Rightarrow$

	Probablili dad
0	$2/6 = 1/3$
1	$2/6 = 1/3$
2	$2/6 = 1/3$

$\Rightarrow p_i = \frac{1}{6} = \frac{1}{2} \cdot \frac{1}{3}$

Si cruz $\left(p = \frac{1}{2}\right)$

x	x mod 3	(x mod 3) + 3
1	1	4
2	2	5
3	0	3
4	1	4
5	2	5
6	0	3

$\Rightarrow$

	Probabilid ad
3	$2/6 = 1/3$
4	$2/6 = 1/3$
5	$2/6 = 1/3$

$\Rightarrow p_i = \frac{1}{6} = \frac{1}{2} \cdot \frac{1}{3}$

$$H = \bar{I} = 6 \cdot \left(\frac{1}{6} \cdot \log_2 \left(\frac{1}{1/6} \right) \right) = \log_2(6) \Rightarrow b)$$

ya que $p_i = \frac{1}{6}$, por lo que equivale a una fuente $\Rightarrow \{0,1,2,3,4,5\}$ con $p_i = \frac{1}{6}$

10) d)

Espacio de las palabras código:

$$4 \text{ mensajes de usuario} \left\{ \begin{bmatrix} 0000000 \\ 0100011 \\ 1011000 \\ \underbrace{11}_{K=2}11011 \end{bmatrix} \right\} 4 \text{ palabras código}$$

ya que: $0100011 \oplus 1011000 = 1111011$

a) $n = k + r$ si es 1-perfecto $\rightarrow n = 2^r - 1 = 2^5 - 1$

$7 = 2 + r$ $n = 31 \Rightarrow$ código $(\overset{n}{31}, \overset{k}{2}) \Rightarrow$ No

$\Rightarrow r = 5$

b) q^K elementos $= 2^K = 4 \Rightarrow k = 2 =$ dimensión $\Rightarrow$ No

c) 1111111 no es palabra código

d) Sí

11) d)

$$\left(\frac{\mathrm{S}}{\mathrm{N}}\right)_m = \frac{E\{a^2\}\cdot x^2(0)}{s_h^2 + ISI} = \frac{5\cdot 1.3^2}{s_h^2 + 5\cdot(0.1^2+0.17^2)} = 10^{1.82} \Rightarrow s_h^2 + 01945 = 0.1279$$

$$\Rightarrow s_h^2 = -0.0666$$

$$ISI = E\{a^2\}\cdot\sum_{n\neq 0} x^2(n) \text{ y } \frac{ISI}{x^2(0)} = DCM = E\{a^2\}\cdot\frac{\sum_{n\neq 0} x^2(n)}{x^2(0)}$$

$$E\{a^2\} = \frac{A^2-1}{3}\cdot d^2 \text{ si los símbolos son equiprobables}$$

12) b)

$$\mathrm{I(bits)} = 10000 \text{ caracteres}\cdot\overline{\mathrm{L}}\frac{\text{bits}}{\text{caracter}} = 21000 \text{ bits}$$

$$\text{ya que } 3000\frac{\text{bits}}{\text{seg}}\cdot 7\text{ seg} = 21000 \text{ bits transmitidos}$$

$$\overline{\mathrm{L}} = 2.1\frac{\text{bits}}{\text{caracter}}$$

$$\mathrm{H} = \sum_{i=1}^{\mathrm{F}} p(i)\cdot\log_2\left(\frac{1}{p(i)}\right) = \left(p\cdot\log_2\left(\frac{1}{p}\right)\cdot F\right) = \frac{1}{F}\cdot F\cdot\log_2(F) = \log_2(F), \text{ ya que } \mathrm{p(i)} = \mathrm{p} = \frac{1}{\mathrm{F}}$$

$$\mathrm{H(F)} = \log_2(F) \quad \text{para símbolos independientes y equiprobables}$$

$$\mathrm{H(F)} \leq \overline{\mathrm{L}} \qquad \mathrm{F} = 2 \rightarrow \mathrm{H(F)} = 1$$

$$\left.\begin{array}{l}\text{Si } \mathrm{F} = 4 \Rightarrow \mathrm{H(F)} = 2\\ \text{Si } \mathrm{F} = 5 \Rightarrow \mathrm{H(F)} = 2.32\end{array}\right\} \Rightarrow \mathrm{F} = 4$$

13) c)

$$\mathrm{d} = 2$$

$$\mathrm{P_E} = \left(\frac{1}{14}\cdot\underset{a1}{\overset{\text{Hay 4}}{\overset{\downarrow}{4}}}\cdot\overset{\#vecinos}{\overset{\downarrow}{2}} + \underset{a3}{\frac{1}{28}\cdot 4\cdot 4} + \underset{a2}{\frac{1}{14}\cdot 8\cdot 3}\right)\cdot Q\left(\frac{d}{s_h}\right) = 2.857\cdot Q\left(\frac{2}{\sqrt{0.48}}\right) = 0.02215$$

14) a)

$$\mathrm{Y_1} \quad 010110$$

$$\left.\begin{array}{l}\mathrm{Y_2} \quad 001011\\ \mathrm{Y_3} \quad 111000\end{array}\right\} \quad \mathrm{Y} = \mathrm{Y_2} + \mathrm{Y_3} = 110011; \quad \mathrm{Z} = 010011$$

$$\mathrm{Z} = \mathrm{Y} + \mathrm{E} \qquad \mathrm{E} = \mathrm{Z} + \mathrm{Y} = 100000 \quad \Rightarrow \text{a)}$$

15) d)

$$\text{probabilidad} = \binom{L}{K} \cdot 0.5^K \cdot (1-0.5)^{L-K} = \binom{L}{K} \cdot 0.5^K \cdot 0.5^{L-K} = \binom{L}{K} \cdot 0.5^L$$

para k = 7 ceros

$$\binom{L}{7} \cdot 0.5^L = \begin{matrix} (\text{para L} = 13) \rightarrow 0.2095 \neq 0.5 \\ (\text{para L} = 15) \rightarrow 0.1964 \neq 0.5 \\ (\text{para L} = 17) \rightarrow 0.1484 \neq 0.5 \end{matrix} \qquad \Rightarrow \text{d)}$$

16) d)

a) $s_i = f(a(i), a(i+1)...., \text{a(i + M)})$

$$\left.\begin{matrix} \#\text{ de } F_0 = A^M = 4 \\ \text{De cada bola, salen A = 4 niveles} \end{matrix}\right\} 4 = 4^M \Rightarrow M = 1$$

$s_{\text{i}} = f(a(i), a(i+1)) \rightarrow \text{No}$

b, c) Es un PAM - 4 con M = 1 ⇒ d)

17) c)

A, B, C → 3 símbolos independientes

$H(F^m) = m \cdot H(F) \rightarrow$ fuente extendida de F símbolos, de orden m - 1

1 = m - 1 ⇒ m = 2

$H(F^2) = 2 \cdot H(F) \Rightarrow$ c)

18) b)

$e = 1, r = 4 \Rightarrow n = 2^r - 1 = 15 \Rightarrow k = n - r = 11$

$\text{Código}(\underset{n}{15}, \underset{k}{11})$

$$P_E = \sum_{j=2}^{15} \binom{15}{j} \cdot p^j \cdot (1-p)^{15-j} \approx \binom{15}{2} \cdot p^2 \cdot (1-p)^{13} = 105 \cdot (10^{-6})^2 \cdot (1-10^{-6})^{13} = 1.05 \cdot 10^{-10}$$

19) a)

$1, D, D^2, D^3, D^4 .....$

a) Cierto

b) si primitivo $\rightarrow L = 2^m - 1 = 31 \rightarrow$ No

c) No lo puedo asegurar

20) d)

a) 0010111

<u>0101110</u>

0111001 $\rightarrow$ no es palabra código $\rightarrow$ No es lineal

b) La distancia entre las 2 últimas palabras código es 2

$d_{min} < 4$ $\qquad$ $d_{min} = 2$

c) $d_{min} \geq 2 \cdot e + 1 \rightarrow e = \left\lfloor \frac{d_{min} - 1}{2} \right\rfloor = \left\lfloor \frac{2-1}{2} \right\rfloor = 0$