Problema 1

ⓐ $ECM = E\{a^2\} \cdot DCM + \cancel{\dfrac{\sigma_{n}^2}{h^2(0)}} \xrightarrow{} 7$ $\Rightarrow$ Minimizar DCM $\equiv$ Minimizar ECM $\Downarrow$ Ecualizador óptimo

sin ruido

$$\begin{pmatrix} R_y(0) & R_y(1) & R_y(2) \\ R_y(1) & R_y(0) & R_y(1) \\ R_y(2) & R_y(1) & R_y(0) \end{pmatrix} \cdot \begin{pmatrix} C_{-1} \\ C_0 \\ C_1 \end{pmatrix} = \begin{pmatrix} R_{ay}(-1) \\ R_{ay}(0) \\ R_{ay}(1) \end{pmatrix} \rightarrow R_y \cdot \phi = R_{ay}$$

$R_y(0) = E\{a^2\} \cdot \rho_x(0) + \sigma_\eta^2$ $\qquad$ $R_{ay}(-1) = E\{a^2\} \cdot x(1)$

$R_y(1) = E\{a^2\} \cdot \rho_x(1)$ $\qquad$ $R_{ay}(0) = E\{a^2\} \cdot x(0)$

$R_y(2) = E\{a^2\} \cdot \rho_x(2)$ $\qquad$ $R_{ay}(1) = E\{a^2\} \cdot x(-1)$

Si $\sigma_\eta^2 = 0 \Rightarrow$
$$\begin{pmatrix} \rho_x(0) & \rho_x(1) & \rho_x(2) \\ \rho_x(1) & \rho_x(0) & \rho_x(1) \\ \rho_x(2) & \rho_x(1) & \rho_x(0) \end{pmatrix} \cdot \begin{pmatrix} C_{-1} \\ C_0 \\ C_1 \end{pmatrix} = \begin{pmatrix} x(1) \\ x(0) \\ x(-1) \end{pmatrix}$$

$\rho_x(0) = 0'3^2 + 0'9^2 + 0'4^2 = 1'06$

$\rho_x(1) = -0'3 \cdot 0'9 - 0'4 \cdot 0'9 = -0'63$

$\rho_x(2) = 0'3 \cdot 0'4 = 0'12$

$$\begin{pmatrix} 1'06 & -0'63 & 0'12 \\ -0'63 & 1'06 & -0'63 \\ 0'12 & -0'63 & 1'06 \end{pmatrix} \cdot \begin{pmatrix} C_{-1} \\ C_0 \\ C_1 \end{pmatrix} = \begin{pmatrix} -0'4 \\ 0'9 \\ -0'3 \end{pmatrix}$$

1. $1'06 \cdot 0'38 - 0'63 \cdot 1'58 + 0'12 \cdot 0'78 = -0'499 \neq -0'4$
2. $1'06 \cdot 0'38 - 0'63 \cdot 1'36 + 0'12 \cdot 0'48 = -0'3964 \simeq -0'4$
3. $1'06 \cdot (-0'38) - 0'63 \cdot 1'36 + 0'12 \cdot (-0'48) = -1'32$

Seguimos con 2. $\rightarrow$ $-0'63 \cdot 0'38 + 1'06 \cdot 1'36 - 0'63 \cdot 0'48 = 0'8998 \simeq 0'9$

$0'12 \cdot (+0'38) - 0'63 \cdot 1'36 + 1'06 \cdot 0'48 = -0'3024 \simeq -0'3$

$\Rightarrow \varphi_2 = (0'38, 1'36, 0'48)$ //

ⓑ $DCM_0 = \dfrac{\sum_{n\neq 0} x^2(n)}{x^2(0)} = \dfrac{0'3^2 + 0'4^2}{0'9^2} = 0'3086$

$DCM_g = \dfrac{\sum'_{n\neq 0} h^2(n)}{h^2(0)} \underset{\downarrow}{=} \dfrac{0'114^2 + 0'066^2 + 0'112^2 + 0'192^2}{0'928^2} = 0'07752$

$h(n) = x(n) * g(n) = (-0'3, 0'9, -0'4) * (0'38, 1'36, 0'48)$

$h(-2) = -0'3 \cdot 0'38 = -0'114$

$h(-1) = 0'9 \cdot 0'38 - 0'3 \cdot 1'36 = -0'066$

$h(0) = -0'4 \cdot 0'38 + 0'9 \cdot 1'36 - 0'3 \cdot 0'48 = 0'928$

$h(1) = -0'4 \cdot 1'36 + 0'9 \cdot 0'48 = -0'112$

$h(2) = -0'4 \cdot 0'48 = -0'192$

$DCM_g = DCM_{min} = \dfrac{ECM_{min}}{E\{a^2\}} = \dfrac{0'36}{5} = 0'072$

También.

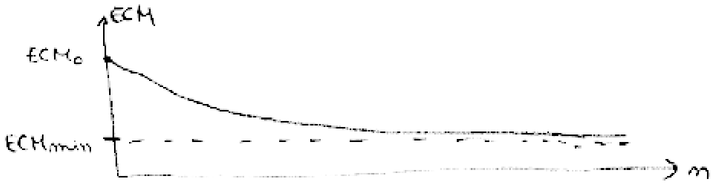
$\dfrac{DCM_0}{DCM_g} = 3'98 \simeq 6\ dB$ //

ⓒ $c^{(n+1)} = \Delta \cdot R_{ay} + (I - \Delta \cdot R_y) \cdot c^{(n)}$ → Iteración determinista

Converge a $\hat{c}$ si $\Delta \leq \Delta_c = \dfrac{2}{\lambda_{max}}$.

$ECM = (c - \hat{c})^T \cdot R_y \cdot (c - \hat{c}) + ECM_{min}$;; $ECM_{min} = E\{a^2\} - R_{ay}^T \cdot \hat{c}$

$\hat{c} = R_y^{-1} \cdot R_{ay}$



También:

$ECM_{min} = E\{a^2\} \cdot DCM_{min} + \cancel{\dfrac{\sigma_{ne}^2}{h^2(0)}} =$

$= 5 \cdot 0'07755 =$

$= 0'3875$ //

$ECM \rightarrow ECM_{min}$ si $\Delta \leq \Delta_c$.

$E\{a^2\} = \dfrac{A^2-1}{3} \cdot d^2 = \dfrac{4^2-1}{3} \cdot 1^2 = 5$

Como $\sigma_{\eta}^2 \simeq \phi \Rightarrow \hat{c} = q_2(n) = (0'38, 1'36, 0'48)$

$$ECM_{final} \simeq ECM_{min} = E\{a^2\} - \underset{\substack{\downarrow \\ R_{ay}(K) = E\{a^2\}\cdot x(-K)}}{R_{ay}^T \cdot \hat{c}} = 5\cdot\left(1 - (-0'4, 0'9, -0'3)\cdot\begin{pmatrix}0'38\\1'36\\0'48\end{pmatrix}\right)$$

$$= 5\cdot\left(1 - 0'928\right) = 0'36 \;//$$

- <u>cálculo de Δ_c:</u>

$$\det(R_y - \lambda_i\cdot I) = \phi \rightarrow \begin{vmatrix} 1'06-\lambda & -0'63 & 0'12 \\ -0'63 & 1'06-\lambda & -0'63 \\ 0'12 & -0'63 & 1'06-\lambda \end{vmatrix} = \phi$$

$$(1'06-\lambda)^3 + 0'63^2\cdot 0'12 + 0'63^2\cdot 0'12 - 0'12^2\cdot(1'06-\lambda) - 0'63^2\cdot(1'06-\lambda) - 0'63^2\cdot(1'06-\lambda) = 0$$

$$(1'06-\lambda)^3 + 0'095 - 0'0144\cdot(1'06-\lambda) - 0'794\cdot(1'06-\lambda) = \phi$$

$$(1'06-\lambda)^3 - 0'8082\cdot(1'06-\lambda) + 0'095 = \phi$$

$$1'19 - 3'37\lambda + 3'18\lambda^2 - \lambda^3 - 0'856 + 0'808\lambda + 0'095 = \phi$$

$$-\lambda^3 + 3'18\lambda^2 - 2'56\lambda + 0'429 = \phi$$

$$\lambda^3 - 3'18\lambda^2 + 2'56\lambda - 0'429 = \phi$$

$$2'33\lambda^3 - 7'41\lambda^2 + 5'97\lambda - 1 = \phi$$

$$\begin{array}{c|cccc} & 2'33 & -7'41 & 5'97 & -1 \\ 1 & & 2'33 & -5 & 1 \\ \hline & 2'33 & -5 & \simeq 1 & \boxed{0} \end{array}$$

$$(\lambda - 1)\cdot(2'33\lambda^2 - 5\lambda + 1) = \phi$$

$$\lambda \simeq 1$$

$$\lambda = \frac{5 \pm \sqrt{25 - 9'32}}{4'66} = \frac{5 \pm 4}{4'66} \simeq \begin{matrix} \nearrow 1'93 \\ \searrow 0'21 \end{matrix}$$

$$\left[\lambda_1 \simeq 0'21, \; \lambda_2 \simeq 1, \; \lambda_3 \simeq 1'93\right]$$

$$\Delta_c = \frac{2}{\lambda_{max}} \simeq \frac{2}{1'93} = 1'036$$

Converge $\forall \ \Delta \leqslant \Delta_c = 1'036$ //

* Resolución exacta (Maple):

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eq:= (1.06-λ)^3 - 0.9082 * (1.06-λ) +0.095 = ϕ
solve(eq, λ);
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$$\lambda_1 = 0'2268 \ , \quad \lambda_2 = 0'9403 \ , \quad \lambda_3 = 2'0128 = \lambda_{max}$$

$\Delta_c = \frac{2}{\lambda_3} = 0'9936 \ \rightarrow$ Converge $\forall \ \Delta \leq \Delta_c = 0'9936$ //

ⓓ

$$P_{E_{PAM-A}} = 2\cdot\left(1-\frac{1}{A}\right)\cdot Q\left[\sqrt{\frac{3\cdot(1+\alpha)}{A^2-1}\cdot\frac{S}{N}}\right] =$$

$$= 2\cdot\left(1-\frac{1}{16}\right)\cdot Q\left[\sqrt{\frac{3\cdot(1+\alpha)}{16^2-1}\cdot\frac{S}{N}}\right] = 1'875\cdot Q\left[\sqrt{\frac{3\cdot 1'5}{255}\cdot S/N}\right]$$

$$¿\alpha_{PAM}?\begin{cases} W_{PAM} = \frac{1+\alpha}{2T} = \frac{1+\alpha}{2}\cdot v_m \Rightarrow 6000 = \frac{1+\alpha}{2}\cdot 8000 \Rightarrow \left[\alpha = 0'5\right] \\ v_t = v_m\cdot q \rightarrow 32000 = v_m\cdot 4 \Rightarrow v_m = 8000 \text{ baudios} \\ q = \log_2 16 = 4 \end{cases}$$

$$P_{E_{QAM-A}} = 4\cdot\left(1-\frac{1}{\sqrt{A}}\right)\cdot Q\left[\sqrt{\frac{3\cdot(1+\alpha)}{A-1}\cdot\frac{S}{N}}\right] =$$

$$= 4\cdot\left(1-\frac{1}{\sqrt{64}}\right)\cdot Q\left[\sqrt{\frac{3\cdot(1'25)}{63}\cdot 10^{2'5}}\right] = 3'5\cdot \underbrace{Q\left[4'338\right]}_{4'098\cdot 10^{-5}} = 1'43\cdot 10^{-4}$$

$$P_{E_{PAM-A}} = P_{E_{QAM-A}}$$

$$Q\Big[\underbrace{\sqrt{0'01765 \cdot S/N}}_{x}\Big] = \frac{1'43 \cdot 10^{-4}}{1'875} = 7'651 \cdot 10^{-5} = \frac{1}{2} \cdot e^{-\frac{x^2}{2}}$$

$$\frac{1'53 \cdot 10^{-4}}{\cancel{0'00162}} = e^{-x^2/2}$$

$$-8'7849 = -\frac{x^2}{2} \Rightarrow x = 4'1916 = 0'01765 \cdot \frac{S}{N}$$

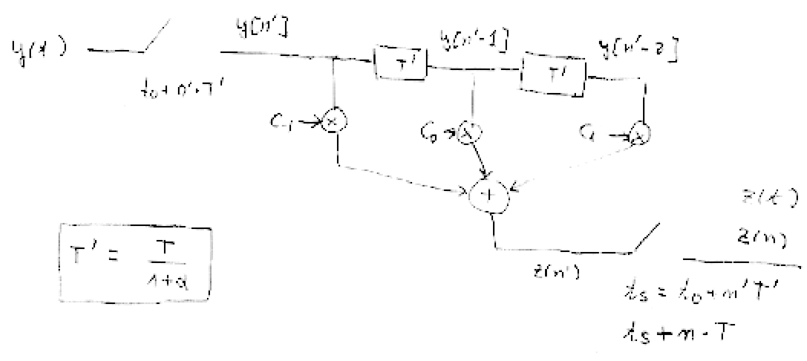
$$\frac{S}{N} = 237'987 = 10^{y}$$

$$y = 2'37 \Rightarrow 23'7 dB = (S/N)_{PAM}$$

$$\Rightarrow C = W \cdot \log_2\left(1 + \frac{S}{N}\right)$$

$$C = 6000 \cdot \log_2(1 + 237'987) = 6000 \cdot \frac{\log 238'987}{\underset{0'30102}{\log 2}} = 47'386 kbps \;//$$

ⓔ $f_m = \dfrac{1}{T'} = \dfrac{1+\alpha}{T} = (1+\alpha)\cdot \vartheta_m \underset{\substack{\alpha = 0'35 \\ \vartheta_m = 2000}}{=} 1'35 \cdot 2000 = 2.700$ Hz //



A partir de z(n'), se regenera z(t) que se muestrea a T.

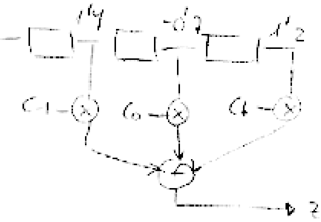
(f) $c^{(0)} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\qquad (-2'5\,,\ 3'1\,,\ 1'4\,,\ -0'7\,,\ 1'2) \rightarrow y(n)$

$$c^{(1)} = c^{(0)} - \Delta \cdot y_n^{*} \cdot e(n)$$

$$h_0(\text{inicial}) = -0'3\cdot 0 + 1\cdot 0'9 + (-0'4)\cdot 0 = 0'9 \Rightarrow \tilde{c}^{(0)} = \frac{c^{(0)}}{h_0} = \begin{pmatrix} 0 \\ 1'1111 \\ 0 \end{pmatrix}$$

$$\Delta_0 = \frac{1}{Le \cdot Ry(0)} = \frac{1}{3 \cdot 3'95} = 0'08438 \qquad \Rightarrow h(0) = 1.$$

$$R_y(0) = \frac{2'5^2 + 3'1^2 + 1'4^2 + 0'7^2 + 1'2^2}{5} = 3'95$$



$$\rightarrow z(n) = 1'4\cdot 0 - 0'7 \cdot \frac{1}{0'9} + 1'2\cdot 0 = -0'7777$$

$\hookrightarrow \hat{a}(n) = -1$

$$e(n) = \frac{z(n)}{h(0)} - \hat{a}(n) = -0'7777 - (-1)$$

$$e(n) = 0'2222$$

$$c^{(1)} = \begin{pmatrix} 0 \\ 1/0'9 \\ 0 \end{pmatrix} - 0'08438 \cdot \begin{pmatrix} 1'4 \\ -0'7 \\ 1'2 \end{pmatrix} \cdot 0'2222 = \begin{pmatrix} -0'02625 \\ 1'1242 \\ -0'02249 \end{pmatrix} //$$

ⓖ $y_{m=0} = y_{m=0'2} - m = (1'2, -3'4, a, 0'8) - 0'2 = (1, -3'2, a-0'2, 0'6)$

$y_{k=1} = (1, -3) * (-0'3, 0'9, -0'4) =$

$$\begin{array}{rrrr} -0'3 & 0'9 & -0'4 & \\ & 0'9 & -2'7 & 1'2 \\ \hline -0'3 & 1'8 & -3'4 & 1'2 \end{array}$$

$y_{k=2} = (-1, 1) * (-0'3, 0'9, -0'4) =$

$$\begin{array}{rrrr} 0'3 & -0'9 & 0'4 & \\ & -0'3 & 0'9 & -0'4 \\ \hline 0'3 & -1'2 & 1'3 & -0'4 \end{array}$$

$\eta_1 = y_{m=0} - y_1 = (1'3, -5, a+2'9, -0'6)$

$\eta_2 = y_{m=0} - y_2 = (0'7, -2, a-1'5, 1)$

$$1'3^2 + 25 + (a+2'9)^2 + 0'6^2 = 0'7^2 + 4 + (a-1'5)^2 + 1$$

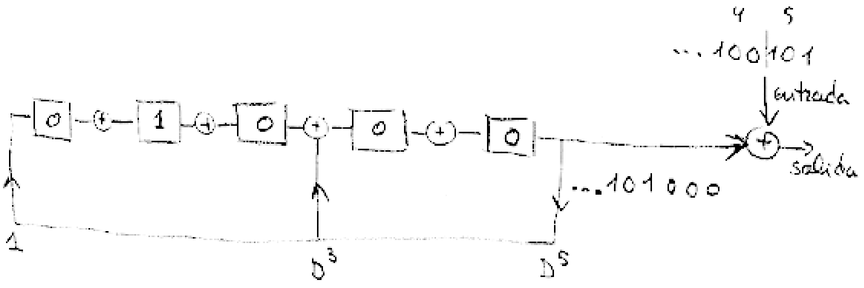
$$1'69 + 25 + \cancel{a^2} + 8'41 + 5'8a + 0'36 = 0'49 + 4 + \cancel{a^2} + 2'25 - 3a + 1$$

$$5'8a + 35'46 = 7'74 - 3a$$

$$8'8a = -27'72$$

$$\boxed{a = -3'15} \quad //$$

ⓗ



Se precisan 5 iteraciones:

$p^{(0)} = 0\ 1\ 0\ 0\ |\ 0$

$p^{(1)} = 0\ 0\ 1\ 0\ |\ 0$

$p^{(2)} = 0\ 0\ 0\ 1\ |\ 0$

$p^{(3)} = 0\ 0\ 0\ 0\ |\ 1$

$p^{(4)} = 1\ 0\ 0\ 1\ |\ 0$

$p^{(5)} = 0\ 1\ 0\ 0\ |\ 1$

↳ salida LFSR

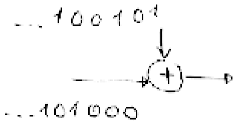
$p^{(3)} = D^4$

$p^{(4)} = D \cdot p^{(3)} \bmod c(D)$

$$\begin{array}{l|l} D^5 & D^5+D^3+1 \\ \hline \underline{D^5+D^3+1} & 1 \\ D^3+1 = p^{(4)}(D) & \end{array}$$

$p^{(5)} = D \cdot p^{(4)} \bmod c(D)$

$$\begin{array}{l|l} D^4+D & D^5+D^3+1 \\ \hline \underline{\emptyset} & \emptyset \\ D^4+D = p^{(5)}(D) & \end{array}$$



$\cdots 00|11|01 \equiv \cdots \boxed{-3\ 1\ -1}$ //

$\left.\begin{array}{l} a \oplus \emptyset = a \\ a \oplus 1 = \bar{a} \end{array}\right\}$

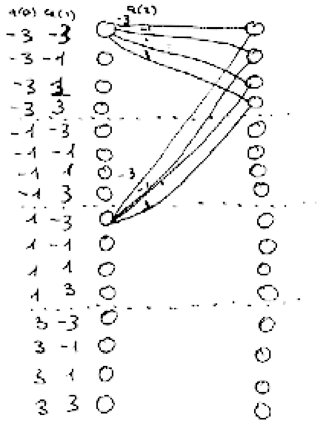
0	00	00	-3
1	01	01	-1
2	10	11	1
3	11	10	3

PAM-4 GRAY

A=4

$M+1=3 \rightarrow M=2$

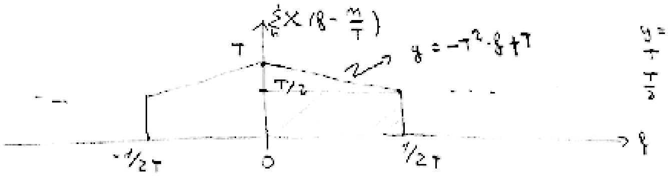
ⓘ $\#F_0 = A^M = 4^2 = 16$;; $\sigma_i(a(i), a(i+1) \cdot a(i+2))$



Problema 2

ⓐ $$DCM = \frac{1}{T\cdot x^2(0)}\cdot\int_{-1/2T}^{1/2T}\left|\sum_n X\left(f-\frac{n}{T}\right)\right|^2\cdot df - 1$$

$$x(0)=\int_{-\infty}^{\infty}X(f)\cdot df = 2\cdot\left(\frac{1}{2T}\cdot\frac{T}{2}+\frac{1}{2}\cdot\frac{1}{2T}\cdot\frac{T}{2}\right)=2\cdot\left(\frac{1}{4}+\frac{1}{8}\right)=\frac{3}{4}$$



$y=ax+b$
$T=b$
$\frac{T}{2}=\frac{a}{2T}+T \rightarrow a=-T^2$

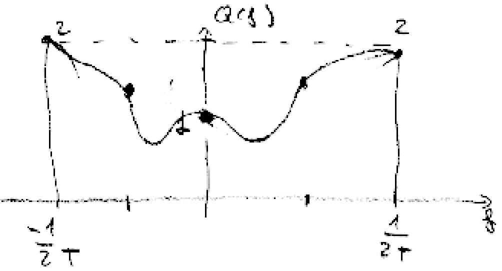
$$\int_{-\frac{1}{2T}}^{1/2T}\left|\sum_n X\left(f-\frac{n}{T}\right)\right|^2\cdot df = 2\cdot\int_0^{1/2T}(-T^2\cdot f+T)^2\cdot df = 2\cdot\int_0^{1/2T}(T^4f^2+T^2-2T^3f)\,df$$

$$=2\cdot\left[T^4\cdot\frac{f^3}{3}+T^2\cdot f-2T^3\frac{f^2}{2}\right]_0^{1/2T}=2\cdot\left[T^4\cdot\frac{1}{24T^3}+\frac{T^2}{2T}-\frac{2T^3}{8T^2}\right]=$$

$$=2\cdot\left[\frac{T}{24}+\frac{T}{2}-\frac{T}{4}\right]=2\cdot T\cdot\frac{7}{24}=\frac{7T}{12}=0'58\hat{3}\,T$$

$$DCM=\frac{1}{T\left(\frac{3}{4}\right)^2}\cdot\frac{7T}{12}-1=\frac{7\cdot 16}{9\cdot 12}-1=0'037 \overset{=\frac{1}{27}}{/\!/}$$

ⓑ $$Q(f)=\frac{T}{\sum_n' X\left(f-\frac{n}{T}\right)}=\frac{T}{-T^2|f|+T}$$



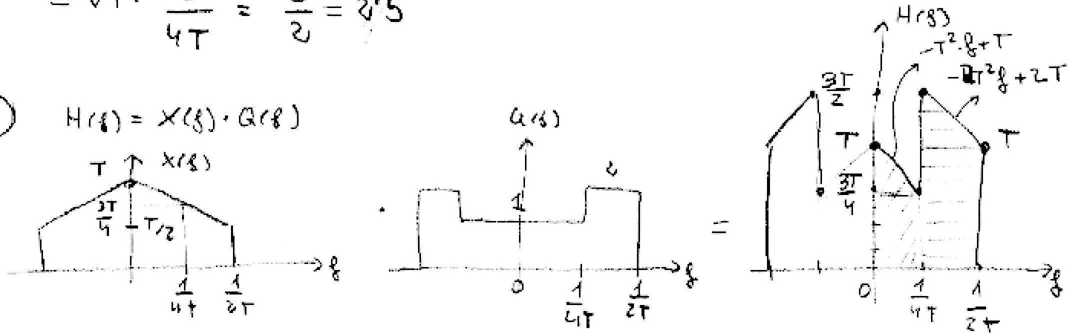
f	Q(f)
$\frac{1}{4T}$	$\frac{T}{-T^2\frac{1}{4T}+T}=\frac{T}{-\frac{T}{4}+T}=\frac{1}{3/4}=\frac{4}{3}=1'3$

ⓒ $$FAR = T\cdot\int_{-1/2T}^{1/2T} |Q(f)|^2 \cdot df = T\cdot 2\cdot\int_{0}^{1/2T} |Q(f)|^2\cdot df =$$

$$= 2T\left[1^2\cdot\frac{1}{4T} + 2^2\cdot\frac{1}{4T}\right] = 2T\left[\frac{1}{4T}+\frac{4}{4T}\right] =$$

$$= 2T\cdot\frac{5}{4T} = \frac{5}{2} = 2'5$$

ⓓ $$H(f) = X(f)\cdot Q(f)$$



$$DCM = \frac{1}{T\cdot h^2(0)}\cdot\int_{-1/2T}^{1/2T}\left|\sum_n H(f-\frac{n}{T})\right|^2\cdot df - 1$$

$$h(0) = \int_{-\infty}^{\infty} H(f)\cdot df = 2\cdot\left(\frac{1}{4T}\cdot\frac{3T}{4} + \frac{1}{2}\cdot\frac{1}{4T}\cdot\frac{T}{4} + \frac{1}{4T}\cdot T + \frac{1}{2}\cdot\frac{1}{4T}\cdot\frac{T}{2}\right) = \overset{=\frac{17}{16}}{1'0625}$$

$$Int. = 2\cdot\int_0^{1/2T}\left|\sum_n H(f-\frac{n}{T})\right|^2 df = 2\cdot\int_0^{1/4T}\underbrace{(-T^2f+T)^2}_{T^4f^2+T^2-2T^3f} df + 2\cdot\int_{\frac{1}{4T}}^{1/2T}\underbrace{(-2T^2f+2T)^2}_{4T^4f^2+4T^2-8T^3f} df =$$

$$= 2\cdot\left[T^4\frac{f^3}{3}+T^2f-2T^3\frac{f^2}{2}\right]_0^{1/4T} + 2\cdot\left[4T^4\frac{f^3}{3}+4T^2f-8T^3\frac{f^2}{2}\right]_{1/4T}^{1/2T} =$$

$$= 2\cdot\left(T^4\frac{1}{3\cdot 4^3\cdot T^3}+\frac{T^2}{4T}-\frac{2T^3}{2}\cdot\frac{1}{16T^2}\right) + 2\cdot\left[\frac{4T^4}{3}\frac{1}{8T^3}+\frac{4T^2}{2T}-\frac{T^3 4}{4T^2}-\frac{4T^4}{3\cdot 4^3\cdot T^3}-\frac{4T^2}{4T}+\frac{T^3\cdot 4}{16T}\right.$$

$$= 2\cdot T\left(\frac{1}{192}+\frac{1}{4}-\frac{1}{16}\right) + 2\cdot T\left(\frac{1}{6}+2-1-\frac{1}{48}-1+\frac{1}{4}\right) =$$

$$= 0'3854T + 0'7916T = 1'177T$$

$$DCM = \frac{1}{T\cdot 1'0625^2}\cdot 1'177T - 1 = 1'0426 - 1 = 0'0426 \;//$$