CONTROL DE TRANSMISIÓN DE DATOS 15-12-00

PROBLEMA 1:

Datos:

$\sigma_\eta^2 = 0.5$

PAM-8 $\rightarrow E\{a^2\} = \frac{A^2-1}{3}\cdot d^2 = \frac{64-1}{3} = \frac{63}{3} = 21$

Siendo d=1, A=8

a) Provocará errores.

Distorsión en los pulsos transmitidos.

$$y[n] = \sum_m a(m)\cdot x[n-m] = a(n)\cdot x(0) + \underbrace{\sum_{m\neq n} a(n)\cdot x[n-m]}_{\text{ISI}}$$

CAG $\Rightarrow \div$ x(0)

$$e(n) = y(n) - a(n) = \frac{1}{x(0)}\cdot\sum_{m\neq n} a(m)\cdot x(m-n)$$

Si |e(n)| > d $\Rightarrow$ Error

b)

$$DP = \frac{\sum_{n\neq 0}|x(n)|}{|x(0)|} = \frac{0.512+0.072}{0.85} = 0.687$$

$$DCM = \frac{\sum_{n\neq 0} x^2(n)}{x^2(0)} = \frac{0.512^2+0.072^2}{0.85^2} = 0.37$$

c)

$$e(n) = \frac{z(n)}{h(0)} - \hat{a}(n)$$

d) ECM_{min}

$$ECM = E\{e^2(n)\} = E\{a^2\}\cdot DCM + \frac{s_{ne}^2}{h^2(0)}$$

$E\{a^2\} = \frac{A^2-1}{3}$, energía de los símbolos enviados por la fuente PAM-8.

σ_{ne}^2, varianza del ruido a la salida del ecualizador.

$\sigma_{ne}^2 = FAR\cdot\sigma_\eta^2$

$$FAR = \frac{\sum_i c_i^2}{h^2(0)}$$

h(n)=x(n) *q(n)

$$DCM = \frac{\sum_{n\neq 0} h^2(n)}{h^2(0)}$$

e)

$$\begin{pmatrix} E(a^2)\cdot f_x(0)+s_h^2 & E(a^2)\cdot f_x(1) & E(a^2)\cdot f_x(2) \\ E(a^2)\cdot f_x(1) & E(a^2)\cdot f_x(0)+s_h^2 & E(a^2)\cdot f_x(1) \\ E(a^2)\cdot f_x(2) & E(a^2)\cdot f_x(1) & E(a^2)\cdot f_x(0)+s_h^2 \end{pmatrix}\cdot\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} E(a^2)\cdot x(1) \\ E(a^2)\cdot x(0) \\ E(a^2)\cdot x(-1) \end{pmatrix}$$

$R_y\cdot\hat{c} = R_{ay}$

$R_y(k) = E\{a^2\}\cdot r_x(k) + s_h^2\cdot d(k)$

$R_{ay}(k) = E\{a^2\}\cdot x(-k)$

$$r_x(0) = 0.512^2 + 0.85^2 + 0.072^2 = 0.989828 \approx 0.989$$
$$r_x(1) = -0.512 \cdot 0.85 + 0.85 \cdot 0.072 = -0.374$$
$$r_x(2) = -0.512 \cdot 0.072 = -0.036864 \approx -0.037$$

$R_y(2) = R_y(-2) = 21 \cdot (-0.037) = -0.777$
$R_y(1) = R_y(-1) = 21 \cdot (-0.374) = -7.854$
$R_y(0) = 21 \cdot 0.989 + 0.5 = 21.269$
$R_{ay}(-1) = 21 \cdot x(1) = 21 \cdot 0.072 = 1.512$
$R_{ay}(0) = 21 \cdot x(0) = 17.85$
$R_{ay}(1) = 21 \cdot x(-1) = 21 \cdot (-0.512) = -10.75$

$$\begin{pmatrix} 21.27 & -7.85 & -0.777 \\ -7.85 & 21.27 & -7.85 \\ -0.777 & -7.85 & 21.27 \end{pmatrix} \cdot \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 1.512 \\ 17.85 \\ -10.75 \end{pmatrix}, \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0.3492 \\ 0.8417 \\ -0.1691 \end{pmatrix}$$

f)

$$s_u^2 = s_h^2 \cdot FAR$$
$$FAR = \frac{\sum_i c_i^2}{h^2(0)}$$

x(-1) x(0) x(1) c_{-1} c_0 c_1
h(n) = x(n) * q(n) = {-0.512, 0.85, 0.072} * {0.3492, 0.8417, -0.1691}
¿h(0)?

$$h(n) = \sum_{i=-1}^{1} c_i \cdot x(n-i) = c_{-1} \cdot x(n+1) + c_0 \cdot x(n) + c_1 \cdot x(n-1)$$
$$h(0) = c_{-1} \cdot x(1) + c_0 \cdot x(0) + c_1 \cdot x(-1) = 0.8272$$
$$FAR = \frac{0.3492^2 + 0.8417^2 + 0.1691^2}{0.8272^2} = 1.2554$$

$\sigma_\upsilon^2 = 0.5 \cdot 1.2554 = 0.6277$

g)

$$\Delta_u \approx \frac{1}{L_e \cdot R_y(0)} = \frac{1}{3 \cdot 7.92} = 0.042$$
$$R_y(0) = \frac{4.2^2 + 0.6^2 + 2.4^2}{3} = 7.92$$
$$c^{(n+1)} = c^{(n)} - \Delta \cdot y_n^* \cdot e(n)$$

z(n) = 4.2 · 0.2 + 0.6 · 0.7 + 2.4 · 0.1 = 1.5
e(n) = z(n) - $\hat{a}$ (n) = z(n)- a(n) = 1.5-3 = -1.5

$$\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.7 \\ -0.1 \end{pmatrix} - 0.042 \cdot \begin{pmatrix} 4.2 \\ 0.6 \\ -2.4 \end{pmatrix} \cdot (-1.5) = \begin{pmatrix} 0.2 \\ 0.7 \\ -0.1 \end{pmatrix} - \begin{pmatrix} -0.2646 \\ -0.0378 \\ 0.1512 \end{pmatrix} = \begin{pmatrix} 0.4646 \\ 0.7378 \\ -0.2512 \end{pmatrix} \xrightarrow[n\to\infty]{} \hat{c}$$

h)

z(n) = 6 · 0.4646 + 4.2 · 0.7378 + 0.6 · (-0.2512) = 5.7356
e(n) = z(n)- $\hat{a}$ (n) = 5.7356 –5 = 0.7356

$$\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0.4646 \\ 0.7378 \\ -0.2512 \end{pmatrix} - 0.042 \cdot \begin{pmatrix} 6 \\ 4.2 \\ 0.6 \end{pmatrix} \cdot 0.7356 = \begin{pmatrix} 0.2792 \\ 0.608 \\ -0.2697 \end{pmatrix} \xrightarrow[n\to\infty]{} \hat{c}$$

PROBLEMA 2:

a) Filtro adaptado → $h_F(t) = g(-t) \Rightarrow x_d(t) = h_F(t) * g(t)$

$H_F(f) = G^*(f) \Rightarrow X_d(f) = H_F(f) \cdot G(f) = G^*(f) \cdot G(f) = |G(f)|^2$

Criterio de Nyquist → $\sum_{n=-\infty}^{\infty} X_d(f - \frac{n}{T}) = T \cdot x_d(0)$

$$\sum_{n=-\infty}^{\infty} |G(f - \tfrac{n}{T})|^2 = T \cdot x_d(0) \equiv cte$$

Siendo, $x_d(0) = \int_{-\infty}^{\infty} X_d(f) \cdot df = \int_{-\infty}^{\infty} |G(f)|^2 \cdot df$

$$x_d(0) = \int_{-\infty}^{\infty} |G(f)|^2 \cdot df = 2\int_0^{1/T} \left(-\sqrt{\tfrac{T}{2}} \cdot T \cdot f + \sqrt{\tfrac{T}{2}}\right)^2 \cdot df$$

$$= 2\int_0^{1/T} \left(\frac{T}{2} \cdot T^2 \cdot f^2 + \frac{T}{2} - T^2 \cdot f\right)^2 \cdot df = 2 \cdot \left[\frac{T^3}{2} \cdot \frac{f^3}{3} + \frac{T}{2} \cdot f - T^2 \cdot \frac{f^2}{2}\right]_0^{1/T}$$

$$= 2 \cdot \left[\frac{T^3}{2} \cdot \frac{1}{3} \cdot \frac{1}{T^3} + \frac{T}{2} \cdot \frac{1}{T} - \frac{T^2}{2} \cdot \frac{1}{T^2}\right] = 2\left[\frac{1}{6} + \frac{1}{2} - \frac{1}{2}\right] = \frac{2}{6} = \frac{1}{3}$$

Por el dibujo ya se ve que $\sum_{n=-\infty}^{\infty} \left|G(f - \frac{n}{T})\right|^2 \neq T \cdot \frac{1}{3}$

No es un pulso de Nyquist.

c)

$$DCM = \frac{1}{T \cdot x^2(0)} \cdot \int_{-1/2T}^{1/2T} \left|\sum_n X(f - \tfrac{n}{T})\right|^2 \cdot df - 1$$

$$x(0) = \int_{-\infty}^{\infty} X(f) df = \frac{1}{2} \cdot \frac{1}{T} \cdot \frac{T}{4} + \frac{2}{T} \cdot \frac{T}{4} = \frac{1}{8} + \frac{1}{2} = \frac{5}{8}$$

$$\int_{-1/2T}^{1/2T} \left|\sum_n X(f - \tfrac{n}{T})\right|^2 \cdot df = 2 \cdot \int_0^{1/2T} \left(-\frac{T^2}{2} \cdot f + \frac{3T}{4}\right)^2 \cdot df = 2 \cdot \int_0^{1/2T} \left(\frac{T^4}{4} \cdot f^2 + \frac{9T^2}{16} - \frac{3T^3}{4} \cdot f\right) df$$

$\frac{T^4}{} \; \frac{f^3}{} \; \frac{T^2}{} \; f \; \frac{T^3}{} \; \frac{f^2}{} \; \Big|_0^{1/2} \quad \frac{T^4}{} \; \frac{}{T^3} \; \frac{T^2}{} \; \frac{}{T} \; \frac{T^3}{}$

$\frac{T}{} \; T \; T \; \frac{T}{}$

$$DCM = \frac{1}{T \cdot (\frac{5}{8})^2} \cdot \frac{19T}{48} - 1 = \frac{64 \cdot 19}{25 \cdot 48} - 1 = \frac{76}{75} - 1 = \frac{1}{75} = 0.015$$

d)

Inversor de canal → $Q(f) = \dfrac{T}{\sum_n X(f - \frac{n}{T})}$

$$Q(f) = \frac{T}{-\frac{T^2}{2} \cdot f + \frac{3T}{4}}$$

f	Q(f)
$\frac{1}{4T}$	1.6
$\frac{1}{8T}$	0.34
$\frac{3}{8T}$	1.77

e)

X(f) · Q(f) = H(f)

$$\sum_n H(f - \tfrac{n}{T}) = \sum_n X(f - \tfrac{n}{T}) \cdot Q(f - \tfrac{n}{T})$$ (Q(f) es periódica 1/T)

$$= Q(f) \cdot \sum_n X(f - \tfrac{n}{T})$$ (inversor de canal)

$$= \frac{T}{\sum_n X(f - \tfrac{n}{T})} \cdot \sum_n X(f - \tfrac{n}{T}) = T$$

$$\Rightarrow \sum_n H(f - \tfrac{n}{T}) = T$$ →Es el criterio de Nyquist

h(n) = δ(n)

⇒ [DCM = 0]