SOLUCIÓN DEL CONTROL DEL 25-5-01

PROBLEMA 1:

a)

$$DCM = \frac{1}{T \cdot x^2(0)} \cdot \int_{-1/2T}^{1/2T} \left| \sum_n X(f - \tfrac{n}{T}) \right|^2 \cdot df - 1$$

$$x(0) = \int_{-\infty}^{\infty} X(f) \cdot df = 2 \cdot \left[T \cdot \frac{1}{2T} + eT \cdot \frac{a}{2T} \right] = 1 + e \cdot a$$

siendo $\frac{1+a}{2T} - \frac{1}{2T} = \frac{a}{2T}$

$$\int_{-1/2T}^{1/2T} \left| \sum X(f - \tfrac{n}{T}) \right|^2 \cdot df = 2\left[\frac{1-a}{2T} \cdot T^2 + \frac{a}{2T} \cdot (T^2 \cdot (e+1)^2) \right] = T \cdot (1 + ae^2 + 2ae)$$

$$DCM = \frac{1}{T \cdot (1+ea)^2} \cdot \left[(1-a) \cdot T + a \cdot T \cdot (e+1)^2 \right] - 1 = \frac{T \cdot (1 + ae^2 + 2ae)}{T \cdot (1+ea)^2} - 1 = \frac{e^2 a \cdot (1-a)}{(1+ea)^2}$$

b) Inversor de canal: h(n)= δ(n)

$$\sum_n H(f - \tfrac{n}{T}) = \sum_n Q(f - \tfrac{n}{T}) \cdot X(f - \tfrac{n}{T}) = Q(f) \cdot \sum_n X(f - \tfrac{n}{T}) = T$$

$$Q(f) = \frac{T}{\sum_n X(f - \tfrac{n}{T})} \Rightarrow \text{DCM} = 0$$

c)

$$Q(f) = T \cdot \frac{1}{T} \cdot \frac{1}{e+1} \cdot \Pi\left(\frac{f}{\frac{1}{T}} \right) + \frac{T}{1-a} \cdot \frac{1-a}{T} \cdot \frac{e}{e+1} \cdot \Pi\left(\frac{f}{\frac{1-a}{T}} \right)$$

$$q(t) = \frac{1}{T} \cdot \frac{1}{e+1} \cdot sinc(\frac{t}{T}) + \frac{e \cdot (1-a)}{T \cdot (e+1)} \cdot sinc\left(\frac{t}{\frac{T}{1-a}} \right)$$

$$a = 0.5 \Rightarrow q(t) = \frac{1}{T} \cdot \frac{1}{e+1} \cdot sinc(\frac{t}{T}) + \frac{e}{2 \cdot T \cdot (e+1)} \cdot sinc\left(\frac{t}{2T} \right)$$

ya que $\quad sinc(\frac{t}{T}) \rightarrow T \cdot \Pi(\frac{f}{\frac{1}{T}})$

$$q(n) = q(t = nT) = \frac{1}{T \cdot (e+1)} \cdot sinc(n) + \frac{e}{2T \cdot (e+1)} \cdot sinc(\frac{n}{2})$$

$$q(0) = \frac{1}{T \cdot (e+1)} + \frac{e}{2T \cdot (e+1)} = \frac{2+e}{2T \cdot (e+1)}$$

sinc(n) = 0, n=1,2,3,4…

n	sinc (n/2) = sen (π·n/2)/(π·n/2)
1	2/π i=0, n=(2i+1)
2	0
3	-2/π·1/3 i=1, n=3
4	0
5	2/π·1/5 i=2, n=5
6	0
7	-2/π·1/7 i=3, n=7

Para n=0 → $q(n) = \frac{2+e}{2T \cdot (e+1)}$

Para n par, n=2i → $q(n) = 0$

Para n impar, n=2i+1 → $q(n) = \frac{e}{2T\cdot(e+1)}\cdot\frac{(-1)^i}{(2i+1)}\cdot\frac{2}{p}$

d)

$$FAR = T\cdot\int_{-1/2T}^{1/2T}|Q(f)|^2\cdot df = T\cdot 2\left[\frac{1-a}{2T}\cdot 1+\frac{a}{2T}\cdot\frac{1}{(e+1)^2}\right]$$

ya que $\frac{1}{2T}-\frac{1-a}{2T}=\frac{a}{2T}$

$$=1-a+\frac{a}{(1+e)^2}$$

si $e\to 0 \Rightarrow FAR\to 1$

$FAR\to(1-a+a)=1\Rightarrow OK!!$

e)

$$P_E = N_u\cdot Q\left(\frac{d}{\sqrt{\frac{s_h^2\cdot FAR}{h^2(0)}+E\{a^2\}\cdot DCM}}\right)$$

Con ecualizador, DCM = 0 $\quad N_\upsilon = 2\cdot\left(1-\frac{1}{A}\right)=2\cdot\left(1-\frac{1}{4}\right)=\frac{3}{2}$

$$P_E=\frac{3}{2}\cdot Q\left(\frac{1}{\sqrt{0.6\cdot 0.9535}}\right)$$ ya que h(n) = δ(n) ⇒h(0) = 1

y $FAR = 1-a+\frac{a}{(1+e)^2}=1-0.5+\frac{0.5}{1.05^2}=0.9535$

$$P_E = 1.5\cdot Q(1.322) = 0.3129$$

PROBLEMA 2:

a)

$s_h^2 = 0.6$

$R_y\cdot c = R_{ay}$

$$\begin{cases} R_y(k)=E\{a^2\}\cdot r_x(k)+s_h^2\cdot d(k)\\ R_{ay}(k)=E\{a^2\}\cdot x(-k)\end{cases}$$

PAM-8 → $E\{a^2\}=\frac{A^2-1}{3}\cdot d^2 = 21$

$\rho_x(0) = 0.74$	$R_y(0) = 16.14$	$R_{ay}(-2) = 2.1$
$\rho_x(1) = 0.32$	$R_y(1) = 6.72$	$R_{ay}(-1) = 4.2$
$\rho_x(2) = 0.04$	$R_y(2) = 0.84$	$R_{ay}(0) = 16.8$
$\rho_x(3) = 0$	$R_y(3) = 0$	$R_{ay}(1) = 4.2$
$\rho_x(4) = -0.001$	$R_y(4) = -0.21$	$R_{ay}(2) = -2.1$

$$\begin{pmatrix} 16.14 & 6.72 & 0.84 & 0 & -0.21\\ 6.72 & 16.14 & 6.72 & 0.84 & 0\\ 0.84 & 6.72 & 16.14 & 6.72 & 0.84\\ 0 & 0.84 & 6.72 & 16.14 & 6.72\\ -0.21 & 0 & 0.84 & 6.72 & 16.14\end{pmatrix}\cdot\begin{pmatrix} c_{-2}\\ c_{-1}\\ c_0\\ c_1\\ c_2\end{pmatrix}=\begin{pmatrix} 2.1\\ 4.2\\ 16.8\\ 4.2\\ -2.1\end{pmatrix}$$

$$\begin{pmatrix} c_{-2} \\ c_{-1} \\ c_0 \\ c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 0.2061 \\ -0.3429 \\ 1.2714 \\ -0.2301 \\ -0.0508 \end{pmatrix} \xrightarrow[h(0)=0.92821]{} \begin{pmatrix} 0.2220 \\ -0.3695 \\ 1.3697 \\ -0.248 \\ -0.00547 \end{pmatrix} \Rightarrow h(0) = 1 \Rightarrow \hat{c} = \begin{pmatrix} 0.2044 \\ -0.3395 \\ 1.2613 \\ -0.2018 \\ -0.10 \end{pmatrix}$$

b)

$$DCM = \frac{\sum_{n \neq 0} x^2(n)}{x^2(0)} = 0.1562$$

$$DCM' = \frac{\sum_{n \neq 0} h^2(n)}{h^2(0)} = 0.01433$$

$$h(n) = (-0.022, 0.081, -0.035, 0.047, 1, 0.026, 0.043, -0.036, -0.0055)$$

c)

$$P_E = N_u \cdot Q\left(\frac{d}{\sqrt{\frac{s_h^2}{x^2(0)} + E\{a^2\} \cdot DCM_1}} \right) = \frac{7}{4} \cdot Q\left(\frac{1}{\sqrt{\frac{0.6}{0.8^2} + 21 \cdot 0.1562}} \right) = \frac{7}{4} \cdot Q(0.4869) \approx 0.777$$

Siendo $N_u = 2 \cdot \left(1 - \frac{1}{A}\right) = \frac{7}{4}$

$$P_E' = N_u \cdot Q\left(\frac{d}{\sqrt{\frac{s_h^2 \cdot FAR}{h^2(0)} + E\{a^2\} \cdot DCM_2}} \right) = \frac{7}{4} \cdot Q\left(\frac{1}{\sqrt{0.6 \cdot 2.127 + 21 \cdot 0.01}} \right) = \frac{7}{4} \cdot Q(0.7964)$$

$$\approx 0.6341$$

$$FAR = \frac{\sum_i ci^2}{h^2(0)} = \sum_i ci^2 = 2.127$$

d)

$c(0) = 1 + D + D^3$ es primitivo

$L = 2^m - 1 = 7 \Rightarrow 33 = 7 \cdot 4 + 5$ iteraciones

$p^{(n)}(d) = p^{(0)}(d) \cdot D^n \bmod c(d)$

$p^{(33)}(d) = p^{(5)}(d) = (1 + D + D^2) \cdot D^5 \bmod c(d) = D + 1 \equiv 110 \rightarrow x(i) = 0$

$$\begin{array}{l|l} D^7 + D^6 + D^5 & \underline{D^3 + D + 1} \\ \underline{D^7 + D^5 + D^4} & D^4 + D^3 + D + 1 \\ \quad D^6 + D^4 & \\ \quad \underline{D^6 + D^4 + D^3} & \\ \qquad D^3 & \\ \qquad \underline{D^3 + D + 1} & \\ \qquad\quad D + 1 & \end{array}$$

e)

$$p^{(34)}(d) = p^{(6)}(d) = D \cdot p^{(5)}(d) \bmod \text{c(d)} = \left(D^2 + D\right) \bmod \text{c(d)} = D^2 + D \equiv 011$$

$$p^{(35)}(d) = p^{(7)}(d) = D \cdot p^{(6)}(d) \bmod \text{c(d)} = \left(D^3 + D^2\right) \bmod \text{c(d)} = D^2 + D + 1 \equiv 11$$

$$\text{x(i)} = 110 = \overset{entero}{6} \rightarrow \overset{PAM-8}{5} \Rightarrow \text{a(n)=5}$$

0	1	2	3	4	5	6	7	entero
-7	-5	-3	-1	1	3	5	7	PAM-8

f)

$$\Delta_v \approx \frac{1}{L_e \cdot R_y(0)} = \frac{1}{5 \cdot 4.546} = 0.044$$

$$R_y(0) = \frac{2.4^2 + 0.2^2 + 1.2^2 + 3.5^2 + 1.8^2}{5} = 4.546$$

g)

$$c^{(1)} = c^{(0)} - \Delta_u \cdot y_n^* \cdot e(n)$$

$$z(n) = 0.1 \cdot 2.4 - 0.2 \cdot 0.2 + 1.2 \cdot (-1.2) - 0.2 \cdot (-0.35) + 0 \cdot 1.8 = -0.54$$

$$a(n) = 5$$

$$e(n) = \frac{z(n)}{h(0)} - a(n) = z(n) - a(n) = -0.54 - 5 = -5.54$$

$$c^{(1)} = \begin{pmatrix} 0.1 \\ -0.2 \\ 1.2 \\ -0.2 \\ 0 \end{pmatrix} - 0.044 \cdot \begin{pmatrix} 2.4 \\ 0.2 \\ -1.2 \\ -3.5 \\ 1.8 \end{pmatrix} \cdot (-5.54) = \begin{pmatrix} 0.68 \\ -0.15 \\ 0.907 \\ -1.053 \\ 0.438 \end{pmatrix}$$

h)

$$c^{(2)} = c^{(1)} - \Delta_u \cdot y_h^* \cdot e(n)$$

$$\Delta_u \approx \frac{1}{L_e \cdot R_y(0)} = \frac{1}{5 \cdot \left(\frac{1.4^2 + 2.4^2 + 0.2^2 + 1.2^2 + 3.5^2}{5}\right)} = 0.046$$

$$z(n) = 1.4 \cdot 0.38 + 2.4 \cdot (-0.17) + 0.2 \cdot 1.059 - 1.2 \cdot (-0.61) - 3.5 \cdot 0.21 = 0.3328$$

$$\hat{a}(n) = 1 \rightarrow e(n) = z(n) - \hat{a}(n) = 0.3328 - 1 = -0.6672$$

$$c^{(2)} = \begin{pmatrix} 0.38 \\ -0.17 \\ 1.059 \\ -0.61 \\ 0.21 \end{pmatrix} - 0.046 \cdot \begin{pmatrix} 1.4 \\ 2.4 \\ 0.2 \\ -1.2 \\ -3.5 \end{pmatrix} \cdot \left(-0.6672\right) = \begin{pmatrix} 0.423 \\ -0.096 \\ 1.065 \\ -0.6468 \\ 0.1025 \end{pmatrix}$$