SOLUCIONES DEL CONTROL DEL 26-5-00

a)

$$P_{E_{SIMB}} = \sum_i P(S_i)\cdot P_{E/S_i} = P(-3)\cdot Q(\tfrac{d}{s_n}) + P(-1)\cdot Q(\tfrac{d}{s_n})\cdot 2 + P(1)\cdot 2\cdot Q(\tfrac{d}{s_n}) + P(3)\cdot Q(\tfrac{d}{s_n})$$

$$= 1.3\cdot Q(\tfrac{d}{s_n}) = 1.3\cdot Q(\tfrac{1}{\sqrt{0.52}}) = 1.3\cdot Q(1.3867) \approx 1.3\cdot 0.1911 = 0.2485$$

b)

$$P_{E_{SIMB}} = N_s\cdot Q\left[\sqrt{\left(\frac{3\cdot(1+a)}{A^2-1}\cdot\frac{S}{N}\right)}\right] \le 0.2485$$

$$\sqrt{\left(\frac{3\cdot(1+a)}{A^2-1}\cdot\frac{S}{N}\right)} \ge 1.3867 \rightarrow \frac{3\cdot 1.5}{15}\cdot\frac{S}{N} \ge 1.9229 \rightarrow \frac{S}{N} \ge 6.4098 \equiv 8.068dB$$

c)

$R_y\cdot \hat{c} = R_{ay}$

$$\begin{cases} R_y(k) = E\{a^2\}\cdot r_x(k) + s_h^2\cdot d(k) \\ R_{ay}(k) = E\{a^2\}\cdot x(-k) \end{cases}$$

$\rho_x(0)=0.2^2+0.8^2+0.2^2=0.72$

$\rho_x(1)=-0.2\cdot 0.8+0.2\cdot 0.8=0$

$\rho_x(2)=-0.2\cdot 0.2=-0.04$

$$E\{a^2\} = P(-3)\cdot(-3)^2 + P(-1)\cdot(-1)^2 + P(1)\cdot(1)^2 + P(3)\cdot(3)^2 = 0.6\cdot 9 + 0.2 + 0.1 + 0.1\cdot 9 = 6.6$$

$OJO: E\{a^2\} = \frac{A^2-1}{3}\cdot d^2 = 5W$ sólo si P(-3)=P(-1)=P(1)=P(3)=1/4

$R_y(2)=R_y(-2)=6.6\cdot(-0.04)=-0.264$

$R_y(1)=R_y(-1)=6.6\cdot 0=0$

$R_y(0)=6.6\cdot 0.72+0.52=5.272$

$R_{ay}(-1)=E\{a^2\}\cdot x(1)=6.6\cdot 0.2=1.32$

$R_{ay}(0)=E\{a^2\}\cdot x(0)=6.6\cdot 0.8=5.28$

$R_{ay}(1)=E\{a^2\}\cdot x(-1)=6.6\cdot(-0.2)=-1.32$

$$R_y\cdot\hat{c} = R_{ay} \rightarrow \begin{pmatrix} 5.272 & 0 & -0.264 \\ 0 & 5.272 & 0 \\ -0.264 & 0 & 5.272 \end{pmatrix}\cdot\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 1.32 \\ 5.28 \\ -1.32 \end{pmatrix}$$

$$\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0.2384 \\ 1.0015 \\ -0.2384 \end{pmatrix} \Rightarrow \text{Luego, ya normalizaré: } \hat{c} \equiv \hat{c}\cdot\frac{1}{h(0)}$$

Por Cramer:

$$\det\begin{vmatrix} 5.272 & 0 & -0.264 \\ 0 & 5.272 & 0 \\ -0.264 & 0 & 5.272 \end{vmatrix} = 146.53 - 0.3674 = 146.1624$$

$$c_{-1} = \frac{\begin{vmatrix} 1.32 & 0 & -0.264 \\ 5.28 & 5.272 & 0 \\ -1.32 & 0 & 5.272 \end{vmatrix}}{146.1624} = \frac{34.8508}{146.1624} = 0.2384$$

$$c_{-2}=\frac{\begin{vmatrix}5.272 & 1.32 & -0.264\\ 0 & 5.28 & 0\\ -0.264 & -1.32 & 5.272\end{vmatrix}}{146.1624}=\frac{146.3842}{146.1624}=1.0015$$

$$c_{-3}=\frac{\begin{vmatrix}5.272 & 0 & 1.32\\ 0 & 5.272 & 5.28\\ -0.264 & 0 & -1.32\end{vmatrix}}{146.1624}=\frac{-34.8508}{146.1624}=-0.2384$$

d)

Sin ecualizador: $$P_E=N_s\cdot Q\left(\frac{d}{\sqrt{\frac{s_h^2+E\{a^2\}\cdot\sum_{n\pm 0}x^2(n)}{x^2(0)}}}\right)$$

$$\text{DCM}_1=\frac{\sum_{n\pm 0}x^2(n)}{x^2(0)}=\frac{0.2^2+0.2^2}{0.8^2}=0.125$$

$$P_E=1.3\cdot Q\left(\frac{1}{\frac{0.52^2}{0.8^2}+6.6\cdot 0.125}\right)=1.3\cdot Q(0.8016)\approx 1.3\cdot 0.3626=0.47139$$

Con ecualizador: $$P_E'=N_s\cdot Q\left(\frac{d}{\sqrt{\frac{s_h^2+FAR}{h^2(0)}+\frac{E\{a^2\}\cdot\sum_{n\neq 0}h^2(n)}{h^2(0)}}}\right)$$

$$\text{FAR}=\sum_i(c_i)^2=0.2384^2+1.0015^2+0.2384^2=1.1167$$

$$h(n)=\sum_{i=-1}^{1}c_i\cdot x(n-i)=c_{-1}\cdot x(n+1)+c_0\cdot x(n)+c_1\cdot x(n-1)$$

$$h(-2)=c_{-1}\cdot x(-1)=0.2384\cdot(-0.2)=-0.04768$$

$$h(-1)=c_{-1}\cdot x(0)+c_0\cdot x(-1)=0.2384\cdot 0.8+1.0015\cdot(-0.2)=-0.00958$$

$$h(0)=c_{-1}\cdot x(0)+c_0\cdot x(-1)+c_1\cdot x(-1)=0.2384\cdot(0.2)+1.0015\cdot(0.8)+(-0.2384)\cdot(-0.2)$$
$$=0.89656$$

$$h(1)=c_0\cdot x(1)+c_1\cdot x(0)=1.0015\cdot 0.2+(-0.2384)\cdot 0.8=0.00958$$

$$h(2)=c_1\cdot x(1)=-0.2384\cdot 0.2=-0.04768$$

coeficientes del ecualizador optimo normalizado:

$$\Rightarrow\left[\hat{c}=\hat{c}\cdot\frac{1}{h(0)}=\frac{1}{0.89656}\cdot\begin{pmatrix}0.2384\\ 1.0015\\ -0.2384\end{pmatrix}=\begin{pmatrix}0.2659\\ 1.1171\\ -0.2659\end{pmatrix}\right]$$

$$P_E' = 1.3 \cdot Q\left(\frac{1}{\sqrt{\frac{0.52 \cdot 1.167}{0.89656^2} + 6.6 \cdot 0.00588}} \right) = 1.3Q(1.14614) \approx 1.3 \cdot 0.25925 = 0.3370$$

vemos que ha disminuido.

$$\text{DCM}_2 = \frac{\sum_{n \neq 0} h^2(n)}{h^2(0)} = \frac{2 \cdot (0.04768^2 + 0.00958^2)}{0.89656^2} = 0.00588$$

e) $\det(R_y - \lambda \cdot I) = 0$

$$R_y = \begin{pmatrix} 5.272 & 0 & -0.264 \\ 0 & 5.272 & 0 \\ -0.264 & 0 & 5.272 \end{pmatrix}$$

$$\begin{vmatrix} 5.272 - l & 0 & -0.264 \\ 0 & 5.272 - l & 0 \\ -0.264 & 0 & 5.272 - l \end{vmatrix} = 0$$

$(5.272-\lambda)^3 - 0.264^2 \cdot (5.272-\lambda) = 0$

$(5.272-\lambda) \cdot [(5.272-\lambda)^2 - 0.264^2] = 0 \quad \rightarrow \lambda_1 = 5.272$

$\rightarrow 27.794 + \lambda^2 - 10.544\lambda - 0.006969 = 0$

$\lambda^2 - 10544\lambda + 27.724 = 0$

$$l = \frac{10.544 \pm \sqrt{0.2799}}{2} = \frac{10.544 \pm 0.59}{2}$$

$\lambda = \rightarrow 5.5365 = \lambda_{max}$

$\rightarrow 5.0074 = \lambda_{min}$

$$\Delta \text{v} = \frac{2}{l_{max} - l_{min}} = \frac{2}{5.5365 + 5.0074} = 0.1897$$

Además, como hay poca dispersión entre autovalores $\Rightarrow \text{s} = \frac{l_{max}}{l_{min}} \approx 1$

$$\Delta_\text{v} \approx \frac{1}{R_y(0)} = \frac{1}{5.272} = 0.1897$$

f) $c^{(n+1)} = \Delta \cdot R_{ay} + (I - \Delta \cdot R_y) \cdot c^{(n)}$

$$c^1 = 0.1897 \cdot \begin{pmatrix} 1.32 \\ 5.28 \\ -1.32 \end{pmatrix} + \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - 0.1897 \cdot \begin{pmatrix} 5.272 & 0 & -0.264 \\ 0 & 5.272 & 0 \\ -0.264 & 0 & 5.272 \end{pmatrix} \right] \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0.2504 \\ 1.0016 \\ -0.2504 \end{pmatrix} + \begin{pmatrix} -9.84 \cdot 10^{-5} & 0 & 0.05 \\ 0 & -9.84 \cdot 10^{-5} & 0 \\ 0.05 & 0 & -9.84 \cdot 10^{-5} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.2504 \\ 1.0016 \\ -0.2504 \end{pmatrix}$$

g) $\text{ECM} = (c - \hat{c})^T \cdot R_y \cdot (c - \hat{c}) + \underbrace{E\{a^2\} - R_{ay}^T \cdot \hat{c}}_{\text{ECM}_{min}}$

- Expresión válida si h(0)=1.
- Si no, $\hat{c}$ se ha de normalizar por h(0).
  $h(n) = \hat{c} \cdot x(n)$.

$$\hat{c} \equiv \hat{c} \cdot \frac{1}{h(0)} = \begin{pmatrix} 0.2659 \\ 1.1171 \\ -0.2659 \end{pmatrix} \Rightarrow h(n) = \sum_{i=-1}^{1} c_i \cdot x(n-i)$$

$h(0)=c_{-1}\cdot x(1)+c_0\cdot x(0)+c_1\cdot x(-1)=0.2659\cdot 0.2+1.1171\cdot 0.8-0.2659\cdot(-0.2)=1.00004\approx 1$

- <u>Inicio</u>

$ECM_1 = ECM$

$$ECM_{min} = E\{a^2\} - R_{ay}^T \cdot \hat{c} = 6.6 - \begin{pmatrix} 1.32 \\ 5.28 \\ -1.32 \end{pmatrix}^T \cdot \begin{pmatrix} 0.2659 \\ 1.1171 \\ -0.2659 \end{pmatrix} = 6.6 - 6.600103 \approx -0.000103$$

$$ECM_1 = \begin{pmatrix} 0-0.2659 \\ 1-1.1171 \\ 0+0.2659 \end{pmatrix}^T \cdot \begin{pmatrix} 5.272 & 0 & -0.264 \\ 0 & 5.272 & 0 \\ -0.264 & 0 & 5.272 \end{pmatrix} \cdot \begin{pmatrix} 0-0.2659 \\ 1-1.1171 \\ 0+0.2659 \end{pmatrix} + ECM_{min}$$

$$= \begin{pmatrix} -1.472 & -0.6173 & 1.472 \end{pmatrix} \cdot \begin{pmatrix} -0.2659 \\ -0.1171 \\ 0.2659 \end{pmatrix} + ECM_{min} = 0.855 - 0.000103 = 0.855$$

$$= \begin{pmatrix} -0.2659 & -0.1171 & 0.2659 \end{pmatrix} \cdot \begin{pmatrix} 5.272 & 0 & -0.264 \\ 0 & 5.272 & 0 \\ -0.264 & 0 & 5.272 \end{pmatrix} \cdot \begin{pmatrix} -0.2659 \\ -0.1171 \\ 0.2659 \end{pmatrix} + ECM_{min}$$

- De otra manera:

$$ECM_1 = E\{a^2\} \cdot DCM + \frac{s_{ne}^2}{x^2(0)} = 6.6 \cdot 0.125 + \frac{0.52}{0.8^2} = 1.6375$$

- <u>Final:</u> $ECM_2 = ECM_{min} = -0.000103$
- Resolución: $10 \cdot \log\left(\frac{ECM_1}{ECM_2}\right) = 10 \cdot \log\left(\frac{0.855}{0.000103}\right) = 39.19 dB$

h)

$$s = \frac{l_{max}}{l_{min}} = \frac{5.5365}{5.0074} = 1.10566$$

$$20 \cdot \log \cdot \left(\frac{s+1}{s-1}\right) = 20 \cdot \log\left(\frac{2.10566}{0.10566}\right) = 20 \cdot \log(19.93) = 25.9893 \frac{dB}{iteración}$$

$25.9893 \frac{dB}{iteración} \cdot x$ iteraciones = 39.19 dB

x = 1.5079 iteraciones

x = 2 iteraciones

En efecto, c^1 del apartado f) ya es casi $\hat{c}$ del apartado c)